On the gravitational waves on the background of anomaly-induced inflation

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Abstract

In the very early Universe matter can be described as a conformal invariant ultra-relativistic perfect fluid, which does not contribute, on classical level, to the evolution of the isotropic and homogeneous metric. However, in this situation the vacuum effects of quantum matter fields become important. The vacuum effective action depends, essentially, on the particle content of the underlying gauge model. If we suppose that there is some desert in the particle spectrum just below the Planck mass, then the effect of conformal trace anomaly is dominating at the corresponding energies. With some additional constraints (which favor extended or supersymmetric versions of the Standard Model rather than the minimal one), one can obtain a stable inflation. In this article we report about the calculation of the gravitational waves in this model. The result for the perturbation spectrum is close to the one for the conventional inflaton model, and is in agreement with the existing Cobe data.

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1 Introduction

Inflation is considered today as a necessary component of the cosmological standard model. Its actual realization can be made through numerous (semi)phenomenological inflaton models (see, for example, [1]). Besides to present a simple solution to the flatness and horizon problems, such issues as metric and density perturbations have been successfully studied in the context of the inflationary model and led to a consistent scenario for structure formation as well as the anisotropies in the relic radiation. These theoretical studies produced sufficient amount of information that could be checked in the observations and experiments, including the recent (and especially future) Cobe data. At the same time, from our point of view, there is a lack of a natural model for inflation. In particular, the inflaton potentials which are necessary to realize the successful inflation are, in the most of the models, postulated

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in some appropriate way. In other words, those are phenomenological potentials, which can be hardly derived from some quantum field theory. On the other hand, the inflaton itself should be some scalar field with the VEV of Planck order. The existence of such field is inconsistent with the modern high energy physics, which favors other candidates for the role of the fundamental theory like the (super)string. Some criticism could be attributed also to the string inflationary models, since they ask for very special initial conditions[2]. Since the universe expands, during inflation, for many orders of magnitude, the typical energy scale greatly decreases, and it is reasonable to look for a model of inflation which should be robust with respect to this change.

An alternative approach to inflation, which satisfies this condition, can be based on the effective action of gravity resulting from the quantum effects of matter fields on the classical gravitational background [3, 5, 4, 6, 7]. The general physical input of this approach is the following [7, 8]. Suppose there is a desert in the spectrum of particles which extends to some orders of magnitude below the Planck scale. Then at these energies the adequate microscopic description is in the framework of effective quantum field theories including all fields of the low-energy effective quantum field theory. This theory can be the Standard Model (SM), or GUT. Due to the existence of the desert, in the very early Universe the matter may be described by the free radiation, that is, microscopically, by the set of massless fields with negligible interactions between them. Due to conformal invariance, these fields decouple from the conformal factor of the metric (we suppose it, for a while, to be isotropic and homogeneous). In this situation the dominating quantum effect is the trace anomaly which comes from the renormalization of the conformal invariant part of the vacuum action. The anomaly-induced effective action can be found explicitly [9, 10] with accuracy to an arbitrary conformal functional which vanishes for the special case of the conformally flat metric [11].

Treating the anomaly-induced action [9, 10] as quantum correction to the Einstein-Hilbert term, one can explore the possibility to have inflationary solutions, investigate their dependence on the initial data and discuss the restrictions coming from the analysis of density and metric perturbations. In this letter we shall concentrate on the later and derive the spectrum of the gravitational waves on the background of the anomaly-induced inflation.

The article is organized as follows. In the next section we present a brief review of the effective action induced by anomaly. In section 3 we discuss the shape of the inflationary background and establish the restrictions on the particle content of the gauge model which produces the vacuum quantum effects. The new aspect, as compared to the previous papers [4, 7], is that we obtain, here, the inflationary solution on the basis of covariant and local version of the effective action. The possible solution of the grace exit problem is also discussed. In section 4 we derive the equation for the metric perturbations. The restrictions on the initial data for the auxiliary fields and metric (vacuum state for the metric perturbations) are implemented using the earlier results for the black hole vacuum in the same anomaly-induced theory. In section 5 the results of numerical analysis of the spectrum are exposed and analyzed. In the last section we draw some conclusions and present discussions including possible future steps in investigating the model.

2 Effective action induced by anomaly

Since we shall need many particular details of the anomaly-induces action, it is reasonable to present its derivation, also in details. The starting point is the action of free massless fields: N_0 scalars (spin-0), $N_{1/2}$ spinors (Dirac, spin-1/2) and N_1 abelian vectors (spin-1). All N's indicate a number of fields (not multiplets). The conformal versions, in curved space-time, for each of these fields are:

$$S_0 = \int d^4x\sqrt{-g}\,\{\,\frac{1}{2}\,g^{\mu\nu}\partial_\mu\,\varphi\partial_\nu\varphi + \frac{1}{12}\,R\varphi^2\,\}\,,$$

$$S_{1/2} = i\int d^4x\sqrt{-g}\,\{\bar{\psi}\gamma^\mu\nabla_\mu\psi\}\,,$$

$$S_1 = \int d^4x\sqrt{-g}\,\{-\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu}\}\,. \tag{1}$$

The only divergences which one meets for the free fields are the one-loop vacuum ones. Using the well-known results for these divergences and the notations

$$C^2 = C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 2\,R_{\alpha\beta}R^{\alpha\beta} + \frac{1}{3}\,R^2\,,$$

for the square of the Weyl tensor, and

$$E = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4\,R_{\alpha\beta}R^{\alpha\beta} + R^2$$

for the integrand of the Gauss-Bonnet topological term, we get

$$\bar{\Gamma}_{div} = -\frac{\mu^{D-4}}{\varepsilon}\int d^Dx\sqrt{-g}\,\{\left(\frac{N_0}{120} + \frac{N_{1/2}}{20} + \frac{N_1}{10}\right)C^2 -$$

$$-\left(\frac{N_0}{360} + \frac{11\,N_{1/2}}{360} + \frac{31\,N_1}{180}\right)E + \left(\frac{N_0}{180} + \frac{N_{1/2}}{30} - \frac{N_1}{10}\right)\Box R\} =$$

$$= -\frac{\mu^{D-4}}{\varepsilon}\int d^Dx\sqrt{-g}\,\{\,wC^2 - bE + c\Box R\,\}\,, \tag{2}$$

where $\varepsilon = (4\pi)^2(D-4)$ is the parameter of dimensional regularization.

The renormalized one-loop effective action has the form

$$\Gamma = S + \bar{\Gamma} + \Delta S\,, \tag{3}$$

where $\bar{\Gamma}$ is the quantum correction to the classical action. ΔS is a local counterterm which is called to cancel the pole in (2). The classical action of the renormalizable theory is $S = S_{matter} + S_{vacuum}$ where S_{vacuum} must include the following structures:

$$S_{vacuum} = \int d^4x\sqrt{-g}\, \left\{a_1 C^2 + a_2 E + a_3 \Box R\right\} + ... \,, \tag{4}$$

where $a_{1,2,3}$ are parameters of this vacuum action. The renormalization of this parameters is necessary element of the consistent quantum theory. If we do not include these terms into the classical action, they will arise anyway due to the quantum corrections and with unremovable divergent coefficients [11][4].

The vacuum action may also include some non-conformal terms like the Einstein-Hilbert one, cosmological term or $\sqrt{-g}R^2$-term, but their renormalization is not necessary in the case of conformal invariant free massless fields which we are dealing with. But they may be, indeed, important from other points of view. In particular, later we shall include the Einstein-Hilbert action into S_{vacuum}.

ΔS in (3) is an infinite local counterterm which is called to cancel the divergent part (2) of $\bar{\Gamma}$. Indeed ΔS is the only source of the noninvariance of the effective action, since naive (but divergent) contributions of quantum matter fields are conformal. The anomalous energy momentum tensor trace is [12, 13]

$$< T_\mu^\mu >= -\frac{2}{\sqrt{-g}}\, g_{\mu\nu}\frac{\delta}{\delta g_{\mu\nu}}\bar{\Gamma} = -\frac{1}{(4\pi)^2}\,(wC^2 - bE + c\Box R)\,, \tag{5}$$

with the same coefficients w, b, c as in (2). The (5) can be also considered as the equation for the finite part of the 1-loop correction to the effective action. The solution of this equation is straightforward [9, 10, 11], and the result looks like

$$\bar{\Gamma} = S_c[\bar{g}_{\mu\nu}] + \frac{1}{(4\pi)^2}\int d^4x\sqrt{-\bar{g}}\,\{a\sigma\bar{C}^2 - b\sigma(\bar{E} - \frac{2}{3}\Box\bar{R}) - 2b\sigma\bar{\Delta}_4\sigma -$$

$$-\frac{1}{12}\,(c - \frac{2}{3}b)[\bar{R} - 6(\bar{\nabla}\sigma)^2 - (\Box\sigma)]^2)\}\,. \tag{6}$$

Here

$$\Delta_4 = \Box^2 + 2\,R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}\,R\Box + \frac{1}{3}\,(\nabla^\mu R)\nabla_\mu$$

is the fourth derivative, conformal invariant and self-adjoint operator. $S_c[\bar{g}_{\mu\nu}]$ is some unknown functional of the metric $\bar{g}_{\mu\nu}(x)$ which serves as an integration constant for any solution of (5). The action (6) includes some arbitrariness, which were extensively investigated recently [16, 14]. Of course, all the arbitrariness is inside the conformal functional $S_c[\bar{g}_{\mu\nu}]$. If one succeeds to rewrite (6) in terms of the original variable $g_{\mu\nu}$, this functional can substituted by an arbitrary conformal-invariant functional of this metric.

[4] The importance of these high derivative terms concerns also any other inflationary model which is going to deal with the quantum fields.

The expression (6) can serve as a basis for the inflationary solution [7] which is identical to the one found by Starobinsky [4] using the (00)-component of the corrected Einstein equations. However, since we are going to consider the perturbations of the metric, it is better to maintain covariance. Therefore one has to rewrite the above solution in terms of original metric. The action (6) can be presented in a nonlocal but covariant form using the original metric and then in a local covariant form via an auxiliary scalar [9]. We shall perform this, following the scheme developed in [14] and applied to the derivation of the Hawking radiation of the black holes in [15]. The non-local form for the anomaly-induced action is

$$\bar{\Gamma} = S_c[\bar{g}_{\mu\nu}] = -\frac{c-\frac{2}{3}\,b}{12(4\pi)^2}\int d^4x\sqrt{-g(x)}\,R^2(x) +$$

$$+\int d^4x\sqrt{-g(x)}\int d^4y\sqrt{-g(y)}\,(E-\frac{2}{3}\Box R)_x\,G(x,y)\,\left[\frac{w}{4}\,C^2-\frac{b}{8}\,(E-\frac{2}{3}\Box R)\right]_y =$$

$$= -\frac{c-\frac{2}{3}\,b}{12(4\pi)^2}\int d^4x\sqrt{-g(x)}\,R^2(x) -$$

$$-\frac{1}{2}\int d^4x\sqrt{-g(x)}\int d^4y\sqrt{-g(y)}\;\frac{\sqrt{b}}{2}\,\left[(E-\frac{2}{3}\Box R)-\frac{w}{b}\,C^2\right]_x \times$$

$$\times G(x,y)\;\frac{\sqrt{b}}{2}\,\left[(E-\frac{2}{3}\Box R)-\frac{w}{b}\,C^2\right]_y +$$

$$+\frac{1}{2}\int d^4x\sqrt{-g(x)}\int d^4y\sqrt{-g(y)}\,\left(\frac{w}{2\sqrt{b}}\,C^2\right)_x\,G(x,y)\,\left(\frac{w}{2\sqrt{b}}\,C^2\right)_y\,. \qquad (7)$$

One has to notice that we have introduced an additional (as compared to [9]) structure $\int C^2\,G\,C^2$ in order to write the first non-local term in the symmetric form. The importance of this term to be included into the conformal part of the effective action $S_c[\bar{g}_{\mu\nu}]$ has been previously discussed in [14, 15] and recently in the last of Ref. [16].

The first term in (7) is local but the last ones are not. However they can be done local through the introduction of the auxiliary scalar fields. Thus we arrive at the following final expression for the anomaly generated effective action of gravity.

$$\bar{\Gamma} = S_c[g_{\mu\nu}] - \frac{c-\frac{2}{3}\,b}{12(4\pi)^2}\int d^4x\sqrt{-g(x)}\,R^2(x) + \int d^4x\sqrt{-g(x)}\,\Big\{\,\frac{1}{2}\,\varphi\,\Delta_4\,\varphi - \frac{1}{2}\,\psi\,\Delta_4\,\psi$$

$$+\varphi\,\left[\frac{\sqrt{b}}{8\pi}\,(E-\frac{2}{3}\,\Box R)\,-\frac{w}{8\pi\sqrt{b}}\,C^2\right] + \frac{w}{8\pi\sqrt{b}}\,\psi\,C^2\Big\}\,. \qquad (8)$$

The last action is classically equivalent to (7), for if one uses the equations for the auxiliary fields φ and ψ, the nonlocal action (7) gets restored. At the same time, local theory (8) is much more useful for the applications. The action (8), exactly as other forms of the anomaly-induced action, contains the arbitrariness related to the conformal invariant functional $S_c[g_{\mu\nu}]$. One has to notice that the only relevant classical term $\int wC^2$ in the classical action of vacuum is conformal invariant and it can be unified with $S_c[g_{\mu\nu}]$. This conformal invariant functional has no importance when one is interested in the behaviour of the conformal factor of the metric. In this case the result (8) is exact one-loop correction to the effective action. At the same time, any other application of (8) requires fixing this arbitrariness in this or that way, so the consideration becomes, in part, phenomenological. For instance, in [15] the conformal functional has been set zero, and this led to the classification of the vacuum states for the semi-classical black holes and to correct, in the leading order, result for the Hawking radiation. Therefore, the $S_c[g_{\mu\nu}] = 0$ choice can serve as a reasonable approximation, and we adopt it here.

The action (8) contains high derivative terms and some remarks are in order. First: our purpose is to investigate the classical equations for the anomaly-induced action. That is why here we do not perform the path integration over the auxiliary fields φ and ψ. As a result the values of the coefficients w, b, c remain unaltered, exactly as it was in the similar black hole application [15], but in contrast to the original work [9]. Second: the kinetic term for the auxiliary field φ is positive while for ψ it is negative. This indicates that these fields should not be considered as physical, but only as auxiliary ones. Third: the very fact that the above action contains higher derivatives does not indicate that it can not be applied to the consistent description of physical phenomena. Let us remind that we do not consider the quantum theory of gravity, and metric here is nothing but the classical background [5]. The higher derivatives show up only in the vacuum part and thus do not jeopardize the unitarity of the quantum theory [6]. In our opinion, for classical theory the only one reasonable criterion of consistency is the existence and stability of the physically acceptable solutions. As we shall see in the next section, the anomaly-induced effective action really produces such solutions. As it was already mentioned above, for the quantum field theory of matter fields in the curved space-time the high derivative classical action of vacuum (4) is necessary, because otherwise the theory is inconsistent [11]. Thus, the appearance of high derivative quantum corrections in (8) does not change this aspect of the vacuum action. The high derivative parts of the vacuum action (classical and quantum) can be very important only in the high energy domain, while at the low energies they should be treated as a very weak correction to the Hilbert-Einstein action. Therefore, the cosmological application of the above action is essentially restricted by the short time after Big Bang, in which the typical energy of matter does not decrease too much. However, in this short time inflation happens, and we consider

[5] One has to notice that the treatment of the corresponding quantum corrections as the $1/N$ approximation to quantum gravity [17, 4, 6] is inconsistent [18].

[6] At the same time, there are certain indications to that even the high derivative quantum theory can be indeed unitary [19]. This was recently found to be true for the particular version of the four-dimensional anomaly-induced theory which we are investigating here [20].

this in the next section.

Before going on in the study of cosmological applications we shall comment on the possibility to reduce the action (6) to the action without the higher derivatives via the properly chosen auxiliary fields. In general, this is possible using the trick suggested in [21] but unfortunately it is not really helpful for the cosmological applications. For instance, introducing two extra scalars χ and θ, one can present (6) in the form

$$\bar{\Gamma} = S_c[\bar{g}_{\mu\nu}] +$$

$$+\frac{1}{(4\pi)^2}\int d^4x\sqrt{-\bar{g}}\,\{w\sigma\bar{C}^2 + b\sigma(\bar{E} - \frac{2}{3}\Box\bar{R}) + 2b\sigma(2\,R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}\,R\Box + \frac{1}{3}\,(\nabla^\mu R)\nabla_\mu)\sigma\} +$$

$$\int d^4x\sqrt{-\bar{g}}\,\{\,-\frac{1}{2}\,\chi^2\,+\,\frac{\sqrt{b}}{2\pi}\,\chi\,\Box\sigma\} +$$

$$\int d^4x\sqrt{-\bar{g}}\,\{\,\frac{1}{2}\,\theta^2\,+\,\frac{\sqrt{3c-2b}}{4\pi}\,\theta\,\Big(\Box\sigma + (\bar{\nabla}\sigma)^2\Big)\}\,. \qquad (9)$$

The last equation resembles the actions of gravity with two scalars (which could be even called inflatons). So, one can try to reduce the study of the cosmological consequences of the above action to the standard procedure of dealing with inflatons. However, this program meets serious difficulties. The most explicit one is that it is non-covariant, and the reduction of the order concerns only σ sector. The tensor degrees of freedom are still with fourth derivatives, and for their reduction one has to introduce tensor auxiliary fields. This means, that there is no explicit way to use the standard inflaton results in our case. At the same time, this presentation indicates to the qualitative similarity between two approaches, and gives some hope that the theory (6) can lead to the predictions not very different from the ones of the phenomenological inflaton models.

3 Inflationary background

In this section we shall consider the inflationary solution for the dynamical equations of the theory with the action

$$S_{total}\;=\;-\,M_P^2\int d^4x\,\sqrt{-g}\,R\,+\,\bar{\Gamma}\,, \qquad (10)$$

where $M_P^2 = 1/16\pi G$ is the square of the Planck mass, and the quantum correction $\bar{\Gamma}$ is taken in the form (8). In what follows we set $S_c[g_{\mu\nu}] = 0$, including to it, when it is not indicated explicitly, also the classical vacuum term.

Since we are going to look for the isotropic and homogeneous solution, the starting point is to choose the metric in the form $g_{\mu\nu} = a^2(\eta)\,\bar{g}_{\mu\nu}$, where η is conformal time. It proves useful to denote, as before, $\sigma = \ln a$. Now, one has to derive the equations for three fields:

φ, ψ, and σ. In the rest of this section we shall consider the conformally flat background and thus set $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$.

The equations for φ, and ψ, have especially simple form

$$\sqrt{-g}\,\left[\Delta_4\,\varphi + \frac{\sqrt{b}}{8\pi}\,(E - \frac{2}{3}\,\Box R)\; - \;\frac{w}{8\pi\sqrt{b}}\,C^2\right] = 0\,,$$

$$\sqrt{-g}\,\left[\Delta_4\,\psi - \frac{w}{8\pi\sqrt{b}}\,C^2\right] = 0\,.$$

One has to remind the transformation law for the quantities which enter the last expression:

$$\sqrt{-g}C^2 = \sqrt{-\bar{g}}\bar{C}^2\,, \qquad \sqrt{-g}\,\Delta_4 = \sqrt{-\bar{g}}\,\bar{\Delta}_4\,, \tag{11}$$

$$\sqrt{-g}(E - \frac{2}{3}\Box R) = \sqrt{-\bar{g}}(\bar{E} - \frac{2}{3}\Box\bar{R} + 4\bar{\Delta}_4\sigma)\,. \tag{12}$$

Taking into account our choice for the fiducial metric $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$, one arrives at the equations

$$\Box^2\,\varphi + \frac{\sqrt{b}}{2\pi}\,\Box^2\sigma = 0\,, \qquad\qquad \Box^2\,\psi = 0\,. \tag{13}$$

The solutions of (13) can be presented in the form

$$\varphi = -\frac{\sqrt{b}}{2\pi}\,\sigma + \varphi_0\,, \qquad\qquad \psi = \psi_0\,. \tag{14}$$

where φ_0, ψ_0 are general solutions of the homogeneous equations $\Box^2\,\varphi_0 = 0\,,\;\; \Box^2\,\psi_0 = 0$. Thus one meets an arbitrariness related to the choice of the initial conditions for the auxiliary fields φ, ψ. But, the inflationary solution does not depend on φ_0, ψ_0. Substituting (14) back into the action and taking variation with respect to σ we arrive at the same equation for σ that follows directly from (6). We shall write this equation in terms of the physical time t, defined, as usual, through $a(\eta)d\eta = dt$. The useful variable is $H(t) = \dot{a}(t)/a(t) = \dot{\sigma}(t)$, since the equation (equivalent to the one of [7]) is of the third order in this variable:

$$\dddot{H} + 7\ddot{H}H + 4\left(1 + \frac{3b}{c}\right)\,\dot{H}H^2 + 4\,\dot{H}^2 + \frac{4b}{c}\,H^4 - \frac{2M_P^2}{c}\,\left(H^2 + \dot{H}\right) = 0\,. \tag{15}$$

It is easy to identify the special solution corresponding to $H = const$:

$$H = \pm\frac{M_P}{\sqrt{b}}\,, \qquad\quad a(t) = a_0\cdot\exp Ht\,. \tag{16}$$

Positive sign corresponds to inflation. The solution (16) was first discovered in [4, 5] and studied in [4, 6] [7].

The detailed analysis shows [4, 7] that the special solution (16) is stable with respect to the variations (not necessary small) of the initial data for $a(t)$, if the parameters of the

[7] In [4] two other similar solutions for the FRW metric with $k = \pm 1$ were found.

underlying quantum theory satisfy the condition $\frac{b}{c} > 0$, that leads, according to (2), to the relation

$$N_1 \,<\, \frac{1}{3}\,N_{1/2} \,+\, \frac{1}{18}\,N_0\,. \tag{17}$$

This constraint is not satisfied for the Minimal Standard Model (MSM) with $N_1 = 12$, $N_{1/2} = 24$ and $N_0 = 4$. However, one can consider some aspects of the neutrino oscillations as an indication that the MSM should be extended, and in this case the inequality (17) can be readily satisfied. Below we consider two versions, each of which leads to stable inflation.

i) Extended SM with $N_1 = 12,\, N_{1/2} = 48,\, N_0 = 8$

and

ii) Supersymmetric MSM with $N_1 = 12,\, N_{1/2} = 32,\, N_0 = 104$.

The advantage of stable inflation is that it occurs independent of the initial data. After the Big Bang, when the Universe starts to expand and the typical energy decreased below the Planck order, we can imagine some kind of "string phase transition". Starting from this point, the effective quantum field theory is an adequate description, and the anomaly-induced model applies. In case of the stable inflation, the initial data for $a(t)$ and its derivatives do not need to be fine tuned, if only the condition (17) is satisfied – the inflation is unavoidable.

Let us calculate the necessary duration of inflation. Suppose we want the Universe to expand in n e-folds. Then the total rate of inflation (in the Planck units [7] of time) will be

$$\frac{a(t_0+\Delta t)}{a(t_0)} = \exp\left\{4\pi\sqrt{\frac{360}{N_{tot}}}\;\Delta t\right\}\,, \qquad N_{tot} = N_0 + 11\cdot N_{1/2} + 62\cdot N_1 \tag{18}$$

so that

$$\Delta\, t = \frac{1}{4\pi}\,\sqrt{\frac{N_t}{360}}\cdot n\,.$$

For the extended and supersymmetric versions of the Minimal Standard Model the time necessary for 65 e-folds is around ten Planck times only. The numerical study has shown that the exponential solution stabilizes, in the theories satisfying (17), in much shorter time. For that reason, one can safely derive the metric perturbations on the exponentially inflating background, independent on the initial data.

The most difficult question is how the inflation ends. So far, we do not have a definite answer to this question, but there are some particular indications that a solution is possible if we take the masses of the matter particles (perfect fluid) into account. We can mention, at the first place, that the first study of the same model, with density of matter $\rho \sim a^{-4}$ inserted into the (00)-component of the equations, has demonstrated the transition to the FRW behaviour at the late time limit [3]. This shows, at least, the possibility of a desirable particular solutions due to the matter fields.

A very important observation is that even when the Universe expands so rapidly as in (16), the gas of matter particles performs some work, and the average energy of these particles decreases. At some instant this energy decreases such that their masses become relevant and then the matter part of the equation has some dust component. It is easy to see that in this case (16) is not anymore a solution. The classical solution for dust $a(t) \sim t^{2/3}$ also is not a solution because of the quantum term. However, in this case both Einstein and matter terms behave like t^{-2} while the "quantum" part behaves like t^{-4}, and very rapidly the anomaly-induced quantum term becomes irrelevant. Thus, one can suppose that at the long-time limit $a(t) \sim t^{2/3}$ is a good approximation for the unknown solution of the equation with matter. Of course, the above consideration is not a solution of the problem, but one can hope that the solution can be found along this line. In the rest of this article we will not discuss the grace exit problem, but instead concentrate on the gravitational waves and their spectrum during the inflationary period.

4 Gravitational wave equations

The equation for the metric perturbations is based on the bilinear expansion of the action of interest (10). Before going on to this cumbersome expansion, let us present the action, through some integrations by parts, in a more convenient form $S = \int d^4x\, L$, with

$$L = \sum_{s=0}^{5} f_s\, L_s = \sqrt{-g}\Big\{ f_0 R + f_1 R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} + f_2 R^{\alpha\beta} R_{\alpha\beta} + f_3 R^2 + f_4 \varphi \Box R + f_5 \varphi \Delta \varphi \Big\} \quad . \tag{19}$$

Here the following definitions have been introduced:

$$\begin{aligned}
f_0 &= -\,M_P^2 \quad ; & (20)\\
f_1 &= a_1 + a_2 + \frac{b-w}{8\pi\sqrt{b}}\,\varphi + \frac{w}{8\pi\sqrt{b}}\,\psi \quad ; & (21)\\
f_2 &= -\,2a_1 - 4a_2 + \frac{w-2b}{4\pi\sqrt{b}}\varphi - \frac{w}{4\pi\sqrt{b}}\,\psi \quad ; & (22)\\
f_3 &= \frac{a_1}{3} + a_2 - \frac{3c-2b}{36(4\pi)^2} + \frac{3b-w}{24\pi\sqrt{b}}\,\varphi + \frac{w}{24\pi\sqrt{b}}\psi \quad ; & (23)\\
f_4 &= -\frac{\sqrt{b}}{12\pi} \quad ; & (24)\\
f_5 &= \frac{1}{2} \quad . & (25)
\end{aligned}$$

Now we have to fix the arbitrariness related to the homogeneous solutions φ_0 and ψ_0 in (14). Let us remind that the choice of initial data for φ and ψ defines the vacuum state for the perturbations. One can make a useful comparison with the vacuum of the black hole background. In the black hole case the vacuum which provides a smooth transition to the Minkowski vacuum at the space infinity is the Boulware one. Let us suppose that the proper cosmological vacuum for the expanding Universe reduces to the Minkowski one at infinite time. For the homogeneous and isotropic solution the equation $\Box^2\,\varphi_0 = 0$ reduces to $\dddot{\varphi}_0 = 0$, and the solution will depend on four integration constants. But, if one requires the finite behaviour at infinite time, the only choice is $\varphi_0 = const$. Now, if requesting the correspondence with the black hole choice at the space infinity [15], the only possibility is to set $\varphi_0 = 0$, and do the same with ψ. Therefore, the consistent choice of the solutions for the auxiliary fields, which we shall use in the rest of the paper, is

$$\varphi = -\frac{\sqrt{b}}{2\pi}\,\sigma\,, \qquad\qquad \psi = 0\,. \tag{26}$$

One could simplify the coefficients f_4 and f_5, substituting their values, but it is better to keep them in order to track back the origin of each term in the final equations.

The calculation proceeds by taking perturbations such that

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}\,, \tag{27}$$

where $g^0_{\mu\nu}$ are background inflationary solutions (16)

$$g^0_{\mu\nu} = (\,1,\ -\delta_{ij}\,e^{Ht}\,)\,, \qquad H = \frac{M_P}{\sqrt{b}}\,, \qquad \mu = 0,1,2,3 \text{ and } i = 1,2,3\,, \tag{28}$$

and $h_{\mu\nu}$ are the perturbations around them. Since we are interested in gravitational waves, we can retain just the traceless and transverse part of $h_{\mu\nu}$, which are the pure tensorial modes [8]. Hence, the metric perturbations are submitted to the restrictions:

$$\partial_i\, h^{ij} = 0\,, \qquad\qquad h_{kk} = 0\,, \tag{29}$$

Besides, the synchronous coordinate condition, $h_{\mu 0} = 0$, is imposed.

The details of the bilinear expansion are postponed to the Appendix. In fact, most of the expansions are identical to the one performed by Gasperini in [24], who has studied the metric perturbations for the second order string-induced action. We have just checked these expansions (and found them absolutely correct). Some other terms are typical of the induced model under consideration and we expanded them for the first time. The final result for the expansions of all structures $L_i f_i$ looks like

$$L_0 \;=\; a^3 f_0\Big\{3H^2h^2 + h\ddot{h} + 4Hh\dot{h} + \frac{3}{4}\dot{h}^2 - \frac{h}{4}\frac{\nabla^2 h}{a^2}\Big\} + \mathcal{O}(h^3) \quad ; \tag{30}$$

$$\begin{aligned} L_1 \;=\;& a^3 f_1\Big\{2H^2\dot{h}^2 - 4H^2h\ddot{h} - 6H^4h^2 - 16H^3h\dot{h} + \ddot{h}^2 + 4H\dot{h}\ddot{h} + \frac{1}{a^4}\nabla^2 h\nabla^2 h + \\ & 2\dot{h}\frac{\nabla^2\dot{h}}{a^2} + (H^2h - 2H\dot{h})\frac{\nabla^2 h}{a^2}\Big\} + \mathcal{O}(h^3) \quad ; \end{aligned} \tag{31}$$

[8] Scalar perturbations in the anomaly-induced model have been first studied in [22] and, in the modern framework, in [23].

$$
\begin{aligned}
L_2 &= a^3 f_2\Big\{ -9H^4h^2 - 24H^3h\dot{h} - 6H^2h\ddot{h} - \frac{9}{4}H^2\dot{h}^2 + \frac{3}{2}H\dot{h}\ddot{h} + \frac{\ddot{h}^2}{4} + \\
&\quad \frac{1}{4a^4}\nabla^2h\nabla^2h - \frac{1}{2}\Big(\ddot{h} + 3H\dot{h} - 3H^2h\Big)\frac{\nabla^2h}{a^2}\Big\} + \mathcal{O}(h^3) \quad ; \qquad (32)
\end{aligned}
$$

$$
L_3 = -12H^2a^3 f_3\Big\{3H^2h^2 + 2h\ddot{h} + 8Hh\dot{h} + \frac{3}{2}\dot{h}^2 - \frac{h}{2}\frac{\nabla^2h}{a^2}\Big\} + \mathcal{O}(h^3) \quad ; \qquad (33)
$$

$$
L_4 = a^3 f_4\Big\{3H\dot{\varphi}\Big[h\ddot{h} + 4Hh\dot{h} + \frac{3}{4}\dot{h}^2 + 3H^2h^2 - \frac{h}{4}\frac{\nabla^2h}{a^2}\Big] + 6H^2h\dot{h}\dot{\varphi}\Big\} + \mathcal{O}(h^3) \quad ; \qquad (34)
$$

$$
L_5 = a^3 f_5\Big\{ -\frac{1}{3}\dot{\varphi}^2h\ddot{h} - \frac{7}{3}H\dot{\varphi}^2h\dot{h} - \frac{7}{4}H^2\dot{\varphi}^2h^2 - \frac{h}{6}\frac{\nabla^2h}{a^2}\dot{\varphi}^2\Big\} + \mathcal{O}(h^3) \quad . \qquad (35)
$$

Disregarding the higher order terms, the equations of motion can be calculated through the Lagrange equation:

$$
\frac{d^2}{dt^2}\frac{\partial L_i}{\partial \ddot{h}} + \nabla^2\frac{\partial L_i}{\partial \nabla^2 h} - \frac{d}{dt}\nabla^2\frac{\partial L_i}{\partial \nabla^2 \dot{h}} - \frac{d}{dt}\frac{\partial L_i}{\partial \dot{h}} - \nabla\frac{\partial L_i}{\partial \nabla h} + \frac{\partial L_i}{\partial h} = 0 \quad . \qquad (36)
$$

The resulting equation for the metric perturbations has the form:

$$
\begin{aligned}
0 &= h\Big[\frac{1}{3}H\dot{\varphi}^2\dot{f}_5 + \frac{1}{2}H^2\dot{\varphi}^2 f_5 + 3H^2 f_0 - H^3(8\dot{f}_1 + 12\dot{f}_2 + 48\dot{f}_3 - 9\dot{\varphi}f_4)\Big] + \\
&+ \dot{h}\Big[H^3(-9f_2 - 36f_3) + \frac{9}{2}\dot{\varphi}H^2 f_4 - 2H\dot{\varphi}^2 f_5 + \\
&+ H^2(12\dot{f}_1 + \frac{3}{2}\dot{f}_2 - 12\dot{f}_3) + \frac{3}{2}H\,(f_0 + \dot{\varphi}\dot{f}_4) - \frac{2}{3}\dot{\varphi}^2\dot{f}_5\Big] + \\
&+ \ddot{h}\Big[H^2(18f_1 + \frac{3}{2}f_2 - 12f_3) + \frac{3}{2}H\dot{\varphi}f_4 - \frac{2}{3}\dot{\varphi}^2 f_5 + H(16\dot{f}_1 + \frac{9}{2}\dot{f}_2) + \frac{1}{2}f_0\Big] + \\
&+ \dddot{h}\Big[H(12f_1 + 3f_2) + 4\dot{f}_1 + \dot{f}_2\Big] + \\
&+ h^{iv}\Big[2f_1 + \frac{1}{2}f_2\Big] - \\
&- \frac{\nabla^2 h}{a^2}\Big[\frac{1}{2}f_0 - H^2(4f_1 + 4f_2 + 12f_3) + \frac{3}{2}H\dot{\varphi}f_4 + \frac{1}{3}\dot{\varphi}^2 f_5 - H(2\dot{f}_1 + \frac{1}{2}\dot{f}_2)\Big] - \\
&- \frac{\nabla^2\dot{h}}{a^2}\Big[H(4f_1 + f_2) + (4\dot{f}_1 + \dot{f}_2)\Big] - \\
&- \frac{\nabla^2\ddot{h}}{a^2}\Big[4f_1 + f_2\Big] + \\
&+ \frac{\nabla^4 h}{a^4}\Big[2f_1 + \frac{1}{2}f_2\Big] \quad . \qquad (37)
\end{aligned}
$$

These are the general equations for the metric perturbations on the background of exponential inflation (16). Now, one has to choose the proper shape of the functions $f_s(t)$, as it was discussed above, and thus obtain the basis for the analysis of these perturbations. Even for the special choice of $f_s(t)$, the equations remain quite difficult to solve analytically, but they admit successful numerical study.

5 The initial data problem and spectrum analysis

A numerical analysis requires some care, mainly in two aspects: the equations must be dimensionless and the initial conditions must be consistently chosen. For the first point, there is no problem in the equation written above: if one sets the Planck mass equal to one, time is automatically measured in the Planck units.

For the initial conditions, we consider that the perturbations have a quantum origin: the seed of the perturbations are quantum fluctuations of the primordial fields. This is fixed in the following way. The equation for gravitational waves are formally equivalent to the equation for a scalar field – the coefficient in front of the tensor mode. Then the perturbations, which originate from the fluctuations of the zero point energy of the quantum fields, have the spectrum characteristic of a scalar quantum field in Minkowski space. This "vacuum state" is well known [13]:

$$h(x,\eta) = h(\eta)\, e^{\pm i\mathbf{n}.\mathbf{x}} \quad , \quad h(\eta) \propto \frac{e^{\pm in\eta}}{\sqrt{2n}} \quad . \tag{38}$$

In these expressions we employed the conformal time, since with it the FRW metric becomes conformal to the Minkowski metric in flat space; $\mathbf{n}$ is the wavenumber vector. Fixing this initial spectrum one can derive how the initial amplitude depends on $\mathbf{n}$. In our case, it becomes

$$h_0 \propto \frac{1}{\sqrt{2n}} \quad , \quad \dot{h}_0 \propto \sqrt{\frac{n}{2}} \quad , \quad \ddot{h}_0 \propto \frac{n^{3/2}}{\sqrt{2}} \quad , \quad \dddot{h}_0 \propto \frac{n^{5/2}}{\sqrt{2}} \quad . \tag{39}$$

In the last expression, the derivatives now are taken with respect to the cosmic time, while the initial conditions are given in terms of the conformal time. Fixing $a_0 = 1$, we are able to take this into account, and the transformation of the initial conditions for the physical time will just introduce constants of order of one. The important thing is that this choice of initial conditions tells us how they depend on the wavenumber value.

Using the dimensionless equation and taking the initial conditions consistent with an initial quantum spectrum, one can integrate the fourth order equation numerically. It is not difficult to plot the final spectrum, but first one has to decide which quantity is interesting to evaluate. A crucial number is the power spectrum of the perturbations. Generally, the solution is given by the function $h(t,\mathbf{x})$. Through a Fourier transformation we get

$$h_{\mathbf{n}}(t) = \frac{1}{(2\pi)^{3/2}}\int h(t,\mathbf{x})\, e^{i\mathbf{n}.\mathbf{x}}\, d^3x \quad . \tag{40}$$

The square of the total amplitude of the gravitational perturbations is obtained through

$$h^2(t) = \int h^2_{\mathbf{n}}(t) d^3n \quad . \tag{41}$$

This can be rewritten as

$$h^2(t) = 4\pi\int h^2_{\mathbf{n}}(t) n^2 dn = 4\pi \int h^2_{\mathbf{n}}(t) n^3 d\ln n = \int P^2_n(t) d\ln n \quad . \tag{42}$$

The quantity $P_n^2(t) = h_{\mathbf{n}}^2(t) n^3$ is called the (square) power spectrum and tells us how the amplitude of the gravitational waves varies in the interval $(n, n+dn)$, which is logarithmic interval in this case. In order to calculate the power spectrum we must square the value for the gravitational perturbation at a given moment of time, for a fixed wavenumber, then take its logarithm and see how it varies with the logarithm of the wavenumber itself. Practically, since we are performing numerical integration, we must plot

$$\ln n^3 h_{\mathbf{n}}^2(t) \times \ln n \, .$$

The power spectrum essentially says how the amplitude of the perturbations depends on their wavelength. A flat spectrum, the Harrison-Zeldovich one, establishes a "democracy principle": all perturbations reenters in the Hubble horizon during the radiative or matter dominated phase (after have left it during the inflationary phase [25]) with the same amplitude independently of their wavelength. One has to notice that, in general, we obtain an expression for the small variations of n and in the long wavelength limit (which are the most important for cosmology) of the type $P_n^2 \propto n^k$. This distribution tells how the amplitude of the perturbations depends on n. The coefficient k is called spectral indice of the perturbation.

In order to compare our results with the traditional ones, consider inflationary scenario based on the Einstein's equations with a cosmological constant. This cosmological constant can arise from an inflaton potential. Then, one meets the deSitter Universe, and the perturbations behave as [26]

$$h(\eta) = \sqrt{\eta}\, c_{\pm}(n) J_{\pm\frac{3}{2}}(n\eta) \quad , \tag{43}$$

where $c_{\pm}$ are integration constants which depends on n. Their dependence on n is fixed by the initial spectrum. If choosing the vacuum state described above, we find that those constants are independent of n. Using the long wavelength limit approximation, $n \to 0$, and taking the dominant mode in the above expression, we find

$$P_n \propto n^{3/2-3/2} = \text{constant} \quad . \tag{44}$$

Hence, the traditional inflationary scenario predicts a flat spectrum, with $k = 0$. It is most relevant, that the analysis of Cobe, Boomerang and Maxima observational programs favor the flat spectrum too [27, 28].

One can use these results to gauge our numerical procedure. Fixing the initial spectrum as above, integrating the equation for the gravitational wave for the traditional inflationary scenario,

$$\ddot{h} - \frac{\dot{a}\,\dot{h}}{a} + \left\{ \frac{n^2}{a^2} - 2\,\frac{\ddot{a}}{a} \right\} h = 0 \quad , \tag{45}$$

with $a(t) = e^{Ht}$, and using the numerical procedure described above, one meets, for the (43) case,

$$k \simeq 0.01 \tag{46}$$

It is easy to see, that this numerical result is close to the analytical one, which is zero. We have considered a variation of n between zero and one. This leads to initial perturbations whose scales are of the order of the Planck length. Applying now this procedure for our model, with $a_1 = a_2 = 0$ and any $\varphi_0 < 1$, and considering the multiplet of the extended Standard Model, we found

$$k \simeq -0.01\,, \tag{47}$$

which is qualitatively in agreement with a flat spectrum. It is important to notice, that this result depends (although not dramatically) on the number of e-folds during the inflationary phase. Here, we have fixed that the inflationary phase lasts ten Planck times, leading to approximately 65 e-folds. These were the same values employed in testing the numerical procedure in the traditional inflationary case exposed above.

Some recent analysis indicates to $-0.15 < k < 0.16$ [27]. Hence, our model predicts a spectral indice different, but not very far from the one predicted by the traditional inflationary scenario. Besides, the prediction has quite a good agreement with the observational results.

Some observation is in order. Our result is essentially based on the number of suppositions about the vacuum state. In particular, we have neglected the conformal invariant functional $S_c[g_{\mu\nu}]$. This may be justified by the obvious success of the similar procedure in the black hole case [15]. At the same time, it is interesting to check whether the dependence on this supposition is really strong. The most natural is to consider the nonzero coefficient a_1 in the classical vacuum action (4) since, from the quantum field theory point of view, it is not possible to set a_1 coefficient in the classical vacuum action (4) exactly zero. The point is that a_1 is subject of renormalization and the consequent renormalization group running. We have performed the corresponding numerical analysis and found that for $a_1 \neq 0$, different results for the spectrum may be obtained. For example, with $a_1 < 1$ and all other entries the same as before, we find the following relation between the spectral indice and the value of a_1:

Table 1

a_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
k	-0.01	-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2

As one can see from this table, when the value of a_1 increases, the spectrum becomes more and more negative. It means that, taking into account a_1, the amplitude of gravitational perturbations decreases as the scale increases. Taking into account the numerical value of the β-function $w/(4\pi)^2 \approx 0.02$ for a_1 in (2) for the extended SM and for the MSSM, we conclude that the admissible value $a_1 < 0.4$ holds under quantum corrections. From the above table, it is possible to see that this value is within the observational limits.

Among other factors, the dependence on the multiplet composition does not seem to be very important. At least, the spectrum is almost the same when one takes the multiplet of the extended Standard Model or the Minimal Supersymmetric Model.

Let us investigate the dependence on the choice of initial data. In order to do this, we substitute two other (arbitrary) sets of initial conditions, different from (39). The first alternative choice is:

$$h_0 \propto 0.001 \quad , \quad \dot{h}_0 \propto 1 \quad , \quad \ddot{h}_0 \propto 1 \quad , \quad \dddot{h}_0 \propto 1 \quad , \tag{48}$$

for any value of n. The numerical analysis shows that these initial conditions lead to the result:

$$k = -0.00002 \quad . \tag{49}$$

The second is

$$h_0 = 0.001 \quad , \quad \dot{h}_0 = -1 \quad , \quad \ddot{h}_0 = 2 \quad \dddot{h}_0 = 0.1 \quad . \tag{50}$$

Then the result for the power spectrum is

$$k \sim -0.00002 \quad . \tag{51}$$

Similar results have arisen for some other choices we tried. Thus, the result is not very sensible to the choice of initial conditions, and one can expect the power spectrum very close to zero. That universality is a very good point, indeed.

The information we have obtained concerns how the amplitude of the perturbations associated with gravitational waves (tensorial perturbations), depends on n, the scale of the perturbation. In the figures 1 and 2 we display the graphics for the multipole coefficients of the two point correlation function of the cosmic microwave background radiation, measured in terms of temperature fluctuation. This coefficient are give by the expression

$$\frac{\Delta T}{T} = \sum_{l=2}^{n} C_l P_l(\cos\theta) \quad , \tag{52}$$

where C_l are the multipole coefficients, $P_l(x)$ is the Legendre polynomial, θ is the angle two observation points in the sky and $\frac{\Delta T}{T}$ is the temperature fluctuation between these two points. The sum begins with $l = 2$ because we exclude the dipole momentum due to the motion of earth.

Using the CMBFast code[9], we plot the spectrum of anisotropy of the Cosmic Microwave Background due to gravitational waves for some of the cases specified above. In figure 1, we use $k = -0.01$, which is the expected prediction with $a_1 = 0$. In figure 2, we fix $k = -0.2$, the prediction with $a_1 = 1$. In plotting this graphics there are essentially two important inputs: the power spectrum k and the matter content of the Universe. The power spectrum

[9]This code has been developed by U. Seljak and M. Zaldarriaga and is avaliable in the website http://www.sns.ias.edu/ matiasz/CMBFAST/cmbfast.html.

is that one calculated in the body of the paper. The matter content is that observed today in the Universe. For the matter content today, we use the most accepted in the literature: 5% of baryonic matter, 35% of cold dark matter and 60% due to a cosmological term. The figures show a small difference in the amplitude of the spectrum due exactly to the fact that the amplitude decreases slight differently in each case.

In principle, the predictions obtained previously agree with the traditional inflationary deSitter phase. In order to have a more detailed description, which could definitely distinguish the predictions of our model from the ones based on the inflaton, one should analyze, in the framework of our approach, the scalar perturbations and the baryogenesis during the inflationary phase. However, the solution of this problem (see [22, 23] for the previous studies) is outside the scope of the present paper, which is devoted to the gravitational waves.

6 Conclusions and Discussions

At very high energies, when the inflation is supposed to occur, the effective mass of the particles may be considered zero, and a conformal description is possible. Then, the quantum effects of the matter fields generate the anomaly-induced effective action for vacuum. In the preceding works [7, 8], the stable inflationary solutions for this anomaly-induced (so called Starobinsky) model was set out. Contrary to the inflaton models, the anomaly-induced inflation comes from the very natural framework, that makes this approach very promising: the degree of phenomenological inputs are much smaller than in the inflaton models. The great advantage of the inflaton models is that they are very well elaborated, and come, with the proper phenomenological suppositions, to the agreement with the available observational data. In order to reduce the gap between the two approaches, in this paper we have investigated the spectrum of the gravitational waves in the anomaly-induced model.

The main difficulty of the anomaly-induced model (exactly the same concerns some other models [29]) is the natural end of inflation. One can suppose, however, that this problem can be solved in the framework of an effective approach. The most promising procedure is to construct real theoretical model for the non-equilibrium behaviour of matter during inflation. It is expected that such a model should describe the slow decrease of the average energy of the particles. Then, from the dimensional reasons and also from the comparison of the time behaviour of the anomaly-induced part of the action, Einstein-Hilbert term and the dust-like matter, one can expect the dynamical mechanism which should end the inflation.

In this paper we concentrated, mainly, on the spectrum of the gravitational perturbations. In the traditional inflationary scenario the spectrum of tensorial perturbations is flat, that is, the amplitude of gravitational waves does not depend on their wavelength (so-called Harrison-Zeldovich spectrum). Strictly speaking, this result applies to the long wavelength regime only. In a quasi-deSitter space-time generated, for example, in the slow roll over models of inflation, a quasi-flat spectrum is generated. The observational results obtained until now agree with a flat or quasi-flat spectrum [27, 28, 30, 31]. In the anomaly-induced framework, the spectrum of gravitational waves is not easy to determine. Even for the exponential inflation background, the equation for the evolution of tensorial perturbations is of the fourth order, with the coefficients strongly depending on the particle content and on the vacuum state. The particle contents considered here were those of the extended SM and of the supersymmetric SM. Even in this case, an analytical solution is not possible for the perturbations of the tensor mode of the metric, and the numerical analysis was performed. The initial conditions for this numerical analysis were suggested by a quantum mechanical mechanism for the generation of perturbations, which is naturally implemented in the context of inflationary phase. The usual expression for the power spectrum was employed and led to the spectral indice compatible with the available observational data on the anisotropy of the relic radiation and very near the values obtained in the inflaton inflationary models.

However, we must stress that it is generally supposed that the observational data coming from the anisotropy of cosmic microwave background radiation are due to density perturbations (which are linked with scalar perturbations); since this density perturbations display a Harrison-Zeldovich (or quasi-Harrison-Zeldovich) spectrum, the tensorial perturbations are supposed to have the same features. In fact, in the standard inflationary scenario the spectral indices for the scalar and tensorial perturbations are connected by the expression $k_T = k_S - 1$ [25]. We remark, however, that the interpretation of these observational data are strongly model-dependent, and only crossing informations from different observational program like CMB anisotropy, high redshift supernova and gravitational lensing, it may be possible to have trustful results for the observational parameters. On the other hand, it is expected that in the near future a direct measurement of the effects of gravitational waves in the anisotropy of cosmic microwave background will be possible, mainly due to polarization of the background photons, and the contribution to CMB anisotropies due to gravitational waves will be separated from the contribution due to density perturbations. Then one will have more chances to distinguish the inflationary model which fits better with these new data.

Many other aspects of the anomaly-induced gravity model presented here deserve further study. Besides the grace exit problem, the fate of density perturbations should also be studied in the present framework. Another interesting aspect is the mechanism of reheating, which can be, hopefully, found along with the grace exit solution. The results obtained so far indicate, in any case, that the anomaly-induced model is a very promising candidate for the description of the very early Universe, especially for obtaining a self-consistent inflationary phase.

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7 Appendix

In order to derive the equation for the gravitational waves, one has to consider the metric perturbations in the Lagrangian and retain the bilinear part of it, so that to get linear terms in the the field equations. In order to compare the technical details, we fix our notations equal to the ones of [24]. In the same paper one can find most of the necessary technical details, like expansions for the components of the curvature tensor. We shall not present these expansions here, and do so only with the φ-dependent terms which one does not meet in [24].

The background solutions are such that

$$g_{00} = 1\,, \qquad g_{ij} = a^2(t)\delta_{ij} = e^{2Ht}\delta_{ij}. \tag{A1}$$

The value of H is constant, and we impose this condition from the beginning just in order to simplify the expressions (actually, our calculation, just as the ones of [24], were performed for an arbitrary H). It is useful to chose, as dynamical variables, the mixed component of the perturbation, which will be denoted as $h^i_j \equiv h$.

The tensor mode of the metric perturbations is defined by the relation

$$g_{\mu\nu} \to g_{\mu\nu} + \delta_{\mu\nu}\,, \qquad \text{where} \qquad \delta_{\mu\nu} = h_{\mu\nu} \tag{A2}$$

parametrized by transverse traceless tensor $h_{\mu\nu}$. Besides we fix the coordinates by the condition $h_{\mu 0} = 0$. Expanding the contravariant components of the metric tensor, at first and second order in h we get

$$\delta^{(1)} g^{\mu\nu} = -h^{\mu\nu}\,, \qquad \delta^{(2)} g^{\mu\nu} = h^{\mu\alpha}h^{\nu}_{\alpha}$$

where $\delta^{(n)}$ denotes the n-th order in the expansion of the corresponding quantity in powers of h. Similarly, for the determinant of metric we get

$$\delta^{(1)}\sqrt{-g} = 0\,, \qquad \delta^{(2)}\sqrt{-g} = -\frac{1}{4}\sqrt{-g}h^{\mu\nu}h_{\mu\nu}\,. \tag{A3}$$

Using the intermediate formula

$$\Box\varphi = \ddot{\varphi} + 3H\,\dot{\varphi} - \frac{1}{2}\,\dot{h}\,h\,\dot{\varphi} + O\left(h^3\right)\,, \tag{A4}$$

we arrive at the following expansions

$$\delta^{(2)}\left(\sqrt{-g}\,(\Box\varphi)^2\right) = a^3\left\{-\frac{1}{4}h^2\left(\ddot{\varphi} + 3H\,\dot{\varphi}\right)^2 - \dot{h}\,h\left(\ddot{\varphi} + 3H\,\dot{\varphi}\right)\dot{\varphi}\right\} \tag{A5}$$

$$\delta^{(2)}\left(\sqrt{-g}R^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi\right)=a^3\ \dot{\varphi}^2\left\{\frac{3}{4}H^2h^2+\frac{1}{2}\ \ddot{h}\ h+\frac{1}{4}\ \dot{h}^2\ +H\ \dot{h}\ h\right\} \tag{A6}$$

$$\delta^{(2)}\left(\sqrt{-g}R\nabla_\mu\varphi\nabla^\mu\varphi\right)=a^3\ \dot{\varphi}^2\left\{3H^2h^2+\ \ddot{h}\ h+\frac{3}{4}\ \dot{h}^2\ +4H\ \dot{h}\ h-\frac{1}{4}h\frac{\nabla^2 h}{a^2}\right\} \tag{A7}$$

These expansions have been used, together with the ones of [24], in the main text of the paper, for the derivation of expressions (35).

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Figure captions

Figure 1: Behaviour of the spectrum of CMB anisotropy for gravitational waves with $k = -0.01$.

Figure 2: Behaviour of the spectrum of CMB anisotropy for gravitational waves with $k = -0.2$.